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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Modal Parameter Identification of Controlled Structures

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Introduction

EVALUATION of the modal parameters of the structural system is essential for the controller design of vibration suppression. The experimental modal analysis methods are well established for open-loop systems.[1] When the structural system is under vibration control and the observation noise is present, the traditional experimental modal test methods based on the fast Fourier transformation cannot be applied.

Recently, a series of the papers were reported,[2–5] which identify the system matrices from closed-loop output data using the eigensystem realization algorithm or the observer/Kalman filter identification.

In this Note, a simple procedure to extract the modal parameters of the closed-loop structural system is proposed. It is formulated by the extended Kalman filter (EKF) technique.[6,7] In several papers[8,9] the EKF technique is used to identify the modal parameters of open-loop structural systems; however, as far as the authors know, there are few papers on the application of the EKF to extract the structural modal parameters directly from the time series of the closed-loop system. Here the unknown parameters consist of eigenfrequencies, damping ratios, mode shapes, and modal masses; thus, the initial values needed in the EKF calculation are supplied by the finite element method (FEM) preanalysis. And the modal model is easily modified by the experimental results because the modal parameters are the common language between the FEM analysis and the experimental modal analysis.

Identification Method

When the structure is subjected to the force $\boldsymbol{u}(t)$, the vibration equation is expressed by the modal coordinate system $\boldsymbol{q}$ as

$$\tilde{\boldsymbol{M}}\ddot{\boldsymbol{q}} + \tilde{\boldsymbol{C}}\dot{\boldsymbol{q}} + \tilde{\boldsymbol{K}}\boldsymbol{q} = \Phi^T \boldsymbol{u} \tag{1}$$

where $\tilde{\boldsymbol{M}}$, $\tilde{\boldsymbol{C}}$, and $\tilde{\boldsymbol{K}}$ are $n \times n$ diagonal matrices whose elements are modal mass, modal damping, and modal stiffness, respectively. Φ is the $p \times n$ modal matrix

$$\Phi = [\phi_1, \ldots, \phi_i, \ldots, \phi_n]$$

and ϕ_i denotes the ith eigenvector normalized to set the modal mass equal to unit. This equation is transformed into the state equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u} \tag{2}$$

where the state vector $\boldsymbol{x}$ consists of $(\boldsymbol{q}, \dot{\boldsymbol{q}})^T$ and $\boldsymbol{\theta}$ is a modal parameter vector defined as

$$\boldsymbol{\theta} = \left\{\zeta_1, \ldots, \zeta_n, \omega_1, \ldots, \omega_n, \phi_1^T, \ldots, \phi_n^T\right\}^T \tag{3}$$

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$\boldsymbol{A}$ and $\boldsymbol{B}$ can be written as

$$\boldsymbol{A}(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \mathrm{diag}\left(-\omega_i^2\right) & \mathrm{diag}(-2\omega_i\zeta_i) \end{bmatrix} \qquad \boldsymbol{B}(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{0} \\ \Phi^T \end{bmatrix}$$

where ω_i and ζ_i are the ith eigenangular frequency and modal damping ratio, respectively.

Assuming the unknown parameter $\boldsymbol{\theta}$ is independent of time and introducing system noise $\boldsymbol{w}_t$ and observation noise $\boldsymbol{v}_t$, the discrete state equation can be written (Fig. 1)

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}_d(\boldsymbol{\theta})\boldsymbol{x}_t + \boldsymbol{B}_d(\boldsymbol{\theta})\boldsymbol{u}_t + \boldsymbol{D}_d(\boldsymbol{\theta})\boldsymbol{w}_t \qquad \boldsymbol{u}_t = \boldsymbol{u}_t^0 + \boldsymbol{u}_t^1 \tag{4}$$

$$\boldsymbol{y}_t = \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{x}_t + \boldsymbol{v}_t \qquad \boldsymbol{C}(\boldsymbol{\theta}) = [\Phi \quad \boldsymbol{0}] \tag{5}$$

where subscript d denotes discretized value with a sampling time Δt, $\boldsymbol{u}_t^0$ is an external force, and $\boldsymbol{u}_t^1$ admissible control force, respectively. Here, $\boldsymbol{y}_t$ is a vector observed by displacement, and $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$ are stationary white noise processes whose mean values and covariance matrices are given by

$$E[\boldsymbol{w}_t] = \boldsymbol{0}, \qquad E[\boldsymbol{v}_t] = \boldsymbol{0}$$

$$E\left[\begin{pmatrix} \boldsymbol{w}_t \\ \boldsymbol{v}_t \end{pmatrix}\begin{pmatrix} \boldsymbol{w}_s^T & \boldsymbol{v}_s^T \end{pmatrix}\right] = \begin{bmatrix} \boldsymbol{Q}_t & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_t \end{bmatrix}\delta_{ts} \tag{6}$$

$$E\left[\boldsymbol{w}_t\boldsymbol{x}_s^T\right] = \boldsymbol{0}, \qquad E\left[\boldsymbol{v}_t\boldsymbol{x}_s^T\right] = \boldsymbol{0} \qquad \text{for} \qquad t \geq s$$

Matrices $\boldsymbol{A}_d$, $\boldsymbol{B}_d$, and $\boldsymbol{D}_d$ are usually obtained numerically; however, in the present case, taking advantage of the modal coordinate expression (2), they are analytically written as

$$\boldsymbol{A}_d = \begin{bmatrix} \boldsymbol{A}_d^1 & \boldsymbol{A}_d^2 \\ \boldsymbol{A}_d^3 & \boldsymbol{A}_d^4 \end{bmatrix}$$

$$\boldsymbol{A}_d^j = \mathrm{diag}\left(a_i^j\right) \qquad \text{for} \qquad j = 1, 2, 3, 4 \quad \text{and} \quad i = 1, 2, \ldots, n$$

$$a_i^1 = e^{-\zeta_i\omega_i\Delta t}\left[\cos\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right) + \frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)\right]$$

$$a_i^2 = \frac{e^{-\zeta_i\omega_i\Delta t}}{\omega_i\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)$$

$$a_i^3 = -\frac{e^{-\zeta_i\omega_i\Delta t}}{\omega_i\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)$$

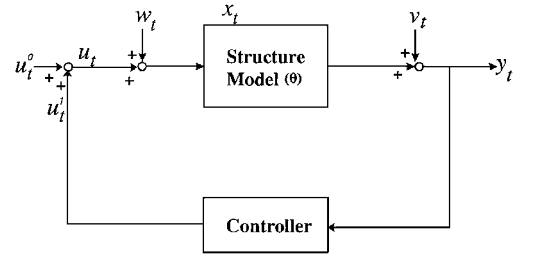

Fig. 1 Closed-loop system.

$$a_i^4 = e^{-\zeta_i \omega_i \Delta t}\left[\cos\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right) - \frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)\right]$$

$$\boldsymbol{B}_d = \begin{bmatrix}\boldsymbol{B}_d^1\\ \boldsymbol{B}_d^2\end{bmatrix} \qquad \boldsymbol{B}_d^1 = \left\{b_{ij}^1\right\}$$

$$\boldsymbol{B}_d^2 = \left\{b_{ij}^2\right\} \qquad \text{for} \qquad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,p$$

$$b_{ij}^1 = \frac{\phi_{ji}}{\omega_i^2}\left\{1 - e^{-\zeta_i\omega_i\Delta t}\left[\cos\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right) + \frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)\right]\right\}$$

$$b_{ij}^2 = \frac{\phi_{ji}}{\omega_i\sqrt{1-\zeta_i^2}}e^{-\zeta_i\omega_i\Delta t}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)$$

$$\boldsymbol{D}_d = \begin{bmatrix}\boldsymbol{D}_d^1\\ \boldsymbol{D}_d^2\end{bmatrix} \qquad \boldsymbol{D}_d^1 = \left\{d_{ij}^1\right\}$$

$$\boldsymbol{D}_d^2 = \{d_{ij}^2\} \qquad \text{for} \qquad i = 1,\ldots,n \quad \text{and} \quad j = 1,\ldots,p$$

$$d_{ij}^1 = \frac{\phi_{ji}}{\omega_i^2}\left\{1 - e^{-\zeta_i\omega_i\Delta t}\left[\cos\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right) + \frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)\right]\right\}$$

$$d_{ij}^2 = \frac{\phi_{ji}}{\omega_i\sqrt{1-\zeta_i^2}}e^{-\zeta_i\omega_i\Delta t}\sin\left(\omega_i\sqrt{1-\zeta_i^2}\Delta t\right)$$

Then let the augmented state-space vector be of the form

$$z_t^T = \left(\boldsymbol{x}_t^T, \boldsymbol{\theta}_t^T\right) \tag{7}$$

The linear stochastic system (4) and (5), including the unknown parameter $\boldsymbol{\theta}$, is expressed by the following nonlinear stochastic system as

$$z_{t+1} = \boldsymbol{f}(z_t, \boldsymbol{u}_t, \boldsymbol{w}_t) \tag{8}$$

$$\boldsymbol{y}_t = \boldsymbol{h}(z_t, \boldsymbol{u}_t, \boldsymbol{v}_t) \tag{9}$$

Now z_t can be estimated by the EKF technique.

Numerical Example

A satellite with flexible solar panels, shown in Fig. 2, is taken as an example. The model is analyzed as a two-dimensional plate-beam structure, and the lower three modes ($n = 3$) are adopted as our identification model. The first and the second modes are bending and the third one is torsional. The exact eigenfrequencies are 1.22, 3.40, and 4.63 Hz, respectively, and the exact damping ratios are 2.0% for all three modes. The actuators are located at the center. And the random response time series at 18 nodes ($p = 18$) are calculated by the mode superposition method setting $\Delta t = 0.05$ s. Therefore, the total number of the unknown modal parameters is $(2 + p) \times n = 60$ [cf. Eq. (3)] in this example. The linear quadratic regulator control law is applied to the model; consequently, the vibration at the tip is suppressed to 1/10.

In the first case, where the system is excited by the external random force u_t^0 with the system noise w_t, the estimated parameters identified at various time $N\Delta t$ (N is time step number) are shown in Table 1. The initial eigenfrequencies $f_i = \omega_i/2\pi$ ($i = 1, 2, 3$) are more than 20% deviating from the exact values, and the initial damping ratios are set to zeros. In this case, w_t is set as

$$\frac{E\left[|w_t|^2\right]}{E\left[\left|u_t^0\right|^2\right]} = O(10^{-4})$$

Table 1 Estimated modal parameters (excited by external force and noise)

	Mode 1		Mode 2		Mode 3	
Parameter	f_1, Hz	ζ_1, %	f_2, Hz	ζ_2, %	f_3, Hz	ζ_3, %
Exact	1.22	2.00	3.40	2.00	4.63	2.00
Initial	0.98	0.00	2.72	0.00	3.71	0.00
$N =$ 100	1.22	1.95	3.42	2.31	4.65	1.95
500	1.22	1.99	3.40	2.04	4.64	2.04
1,000	1.22	1.98	3.40	2.02	4.63	2.02
20,000	1.22	2.00	3.40	2.00	4.63	2.00

Table 2 Estimated modal parameters (excited by noise)

	Mode 1		Mode 2		Mode 3	
Parameter	f_1, Hz	ζ_1, %	f_2, Hz	ζ_2, %	f_3, Hz	ζ_3, %
Exact	1.22	2.00	3.40	2.00	4.63	2.00
Initial	0.98	0.00	2.72	0.00	3.71	0.00
$N =$ 100	1.18	0.04	3.45	0.56	4.66	1.51
1,000	1.24	0.74	3.42	1.28	4.65	2.09
5,000	1.22	1.01	3.40	1.83	4.63	2.02
48,000	1.22	1.87	3.40	1.90	4.63	2.01
150,000	1.22	2.00	3.40	2.00	4.63	1.99

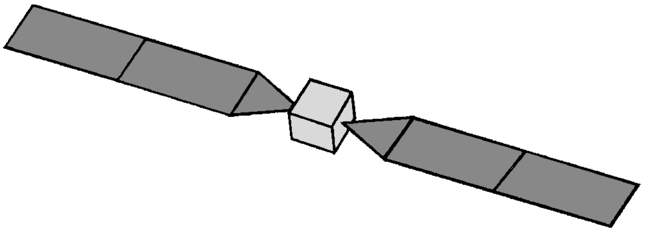

Fig. 2 Numerical calculation model.

The eigenfrequencies converge to the exact values at $N = 1000$, and for the estimated damping ratios, whose initial values are zeros, reasonable agreement with the exact values is apparent, though $N = 20{,}000$ is needed to converge within 0.3% error for the first damping ratio.

In the second case, where the system is excited only by the noise w_t, the estimated values are also shown in Table 2. This is possibly the worst case, and the convergence is not as good as in the first case, especially for the damping ratios. Nonetheless, the EKF technique gives almost the same parameters with the exact value after a sufficient number of time steps.

Conclusion

The EKF technique is applied to extract the modal parameters of the controlled structures. In the calculation, the initial values for the EKF can be supplied either by the finite element analysis or by the open-loop vibration test. In the numerical examples, though the initial values for the damping ratios are set to zero, the extracted results exhibit a good convergence and accuracy for all of the modal parameters.

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Pilot Prefilter Design via H_∞ Model-Matching Approach

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Introduction

In recent years the development of relaxed static stability airplanes has required that flight control systems provide higher levels of augmentation for rapid and precise response. Providing acceptable pilot-in-the-loop handling qualities is a critical design criteria for these highly augmenting control systems.[1–3] Many modern fighter aircraft utilize pilot prefilters, which compensate the pilot's command to improve handling qualities to a satisfactory level while maintaining the excellent stability properties provided by feedback controller design. The idea of separating feedback design from prefilter design, which has been around for more than a decade, is to simultaneously achieve excellent stability properties and desired pilot-in-the-loop behavior.[4,5]

A conventional optimization approach to pilot prefilter design employs a quadratic optimization technique to minimize the model-matching error between the ideal pilot-aircraft dynamics and the actual dynamics. Techniques other than optimization approaches also have been used in the past for the design of the prefilter.[6] The purpose of this Note is to propose an H_∞ model-matching method to design a pilot prefilter with feedforward compensation capability. Model-matching via H_∞ criterion is a worst-case design, which considers the maximum error between the ideal model and the actual model, while for quadratic criterion, the mean-square error is considered. In addition to the numerous successful applications of the H_∞ control theory to feedback controller design for flight control systems, it is shown here that the standard H_∞ framework also provides a very simple and efficient approach to pilot prefilter design.

Problem Formulation

An experimental aircraft developed by Aeronautical Industry Development Center (AIDC) is considered here. The longitudinal motion of the aircraft at the flight condition of 0.8 Mach number and 10,000-ft altitude is described by the following state-space model:

$$\begin{bmatrix} \dot\alpha \\ \dot u \\ \dot\theta \\ \dot q \end{bmatrix} = \begin{bmatrix} -2.191 & -0.0005 & 0 & 0.9940 \\ -5.2400 & -0.0218 & -32.17 & -0.0001 \\ 0 & 0 & 0 & 1 \\ 4.600 & -0.0006 & 0 & -1.0100 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} -0.2611 & -0.150 & 0.0397 \\ 23.20 & -15.30 & -24.90 \\ 0 & 0 & 0 \\ -31.10 & 215.4 & -4.170 \end{bmatrix} \begin{bmatrix} \delta_H \\ \delta_{\mathrm{LEF}} \\ \delta_{\mathrm{TEF}} \end{bmatrix} \tag{1}$$

and the measurement equation is

$$\begin{bmatrix} N_Z \\ q \\ \alpha \\ \theta \end{bmatrix} = \begin{bmatrix} 60.70 & 0.0010 & 0 & -0.2770 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} -5.730 & 4.120 & -2.770 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_H \\ \delta_{\mathrm{LEF}} \\ \delta_{\mathrm{TEF}} \end{bmatrix} \tag{2}$$

where α, u, θ, q, and N_Z are the angle of attack (degrees), velocity along the longitudinal axis (feet per second), pitch angle (degrees), pitch rate (degrees per second), and normal acceleration (feet per second squared), respectively. The proposed control configuration is depicted in Fig. 1. There are three control surfaces used in the longitudinal motion, namely, elevator (δ_H), leading-edge flap (δ_{LEF}), and trailing-edge flap (δ_{TEF}). The bandwidth of the three surface actuators is 4 Hz. The elevator deflection signal comes from the output of the H_∞ feedback controller $K(s)$, which was described in Ref. 7. The LEF/TEF deflections are trim controls and are scheduled with Mach number M and angle of attack α, providing the optimal lift/drag ratio. In the form of transfer function, the aircraft dynamics can be expressed alternatively as

$$\begin{bmatrix} N_z(s) \\ q(s) \\ \hdashline \alpha(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \delta_H(s) \\ \hdashline \delta_{\mathrm{LEF}}(s) \\ \delta_{\mathrm{TEF}}(s) \end{bmatrix} \tag{3}$$

The feedback controller $K(s) = [K_1(s) \;\; K_2(s)]$ that employs the measurements

$$\begin{bmatrix} N_z \\ q \end{bmatrix}$$

to generate the elevator deflection $\delta_H = K_1(s)N_z + K_2(s)q$ has been designed[7] to compensate for the effect of c.g. shift and to satisfy the following specifications. 1) Short-period damping ratio ζ_{sp}

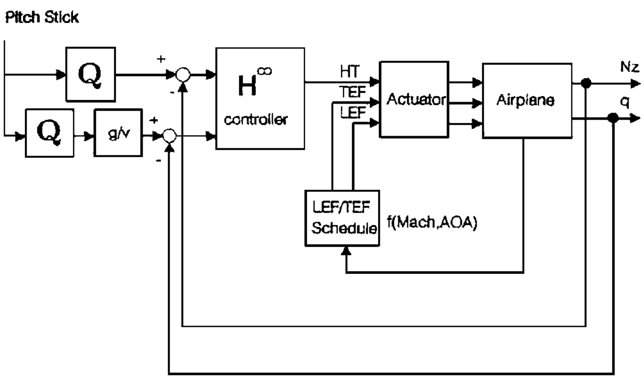

Fig. 1 Longitudinal control configuration.

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